Anger, Sadness and Bear Markets.

Defining and Identifying

Bear Markets

Robert B. Durand*
Marta Simon
Alex Szimayer

JEL classification: G12

Keywords: behavioural finance, affective states, bear markets.

Robert B. Durand is from the University of Western Australia, M250 (Accounting and Finance), 35 Stirling Highway, Crawley, Western Australia, 6009. Marta Simon is from Asgard Capital Management, Sydney, Australia. Alex Szimayer is from the School of Finance & Applied Statistics, the Australian National University, ACT 0200. We gratefully acknowledge the comments made by delegates to the Asian Finance Association Conference held in Kuala Lumpur in July 2005. Additional material referred to in the paper may be found on Robert Durand’s website http://www.biz.uwa.edu.au/home/our_staff/staff_list/durand_robert
* Corresponding author: Phone + 61 8 6488 3764. Fax + 61 8 6488 1047.
E-mail: Robert.Durand@uwa.edu.au
Defining and Identifying Bear Markets

Robert B. Durand
Marta Simon
Alex Szimayer

There is no generally accepted, formal definition of what constitutes a bear or a bull market. This is surprising, considering how often the terms are used to refer to stock market cycles. A typical definition is provided by the Australian Stock Exchange: a bull market occurs when “share prices are generally rising”, while a bear market occurs when “share prices are falling quite sharply and experts expect further falls”.\(^1\) There are a number of methodological difficulties associated with such a definition. To start with they do not prescribe what market index should be used to measure the decline or increase in share prices. Measuring the timing of bear/bull markets and the amount of loss suffered or gained by investors in these states differs greatly, depending on whether the stock market index used in the analysis is a price index or a total return index, equal-weighted or value-weighted, adjusted for inflation/dividend or not. Second, the definitions are vague in terms of the minimum length of time a bear/bull market should last for. Does the expression “prolonged period” refer to 1 month, 2 months or a year? The issue is particularly important when trying to determine whether a stock market crash can be classified as a bear market.\(^2\) The concept of negative expected returns in the ASX definition of bear market is also unsuitable when measuring bear markets simply because expectations cannot be observed ex ante. According to Fama (1970), if price sequence of securities follows a martingale (or a sub-martingale) type process, then expected returns cannot be negative.\(^3\)

---

2. Categorising a stock market crash as a bear market may lead to the inclusion of potentially anomalous return observations in the analysis.
3. According to the martingale model, the expected value of a security at any future time is equal to its value today.
Two main approaches have been adopted in the finance literature to analyze the state of the stock market. The first one advocates a comparison between the return on a market index and a critical threshold value to separate stock market conditions into bear and bull phases (Fabozzi and Francis, 1977; Kim and Zumwalt, 1979; Wiggins, 1992; Bhardwaj and Brooks, 1993).

The second approach to identifying stock market cycles is called the trend-based approach. This approach has been developed as a consequence of the direct applicability of the business cycle techniques to analyze the cyclical behaviour of stock prices. There are two main techniques that can be viewed as a trend-based approach to categorizing stock market conditions. The first, pioneered by Hamilton (1989), considers a parametric specification of the data generating process of the variable. It has been used by Maheu and McCurdy (2000) to detect bull and bear market phases in stock returns. The second technique takes a non-parametric perspective by focusing more on the specific features of the original data series.4

The pattern recognition algorithm employed in the current research is based on the works of Csörgő and Horvath (1997), Kühn (1999) and Chen and Gupta (2000). To our knowledge, our approach has not been used previously to identify stock market cycles. Its advantage over other approaches is that our technique does not involve imposing an arbitrary structure on security returns. The method adopted in this study tests whether structural breaks have occurred somewhere in the sample and, if so, estimates the time of their occurrence. It involves calculating least squared estimates (LSE) of the location of change points in the mean of the return series and choosing those points where the LSE is minimized. Since the number of change points is unknown, we need to adopt a model selection process that ranks the alternative models and identifies the one that best fits the data. To achieve this, we use the Bayesian Information Criterion (BIC) of Schwarz (1978) to determine the optimal trade-off between model complexity and the model’s ability to accurately represent the data. The estimated optimal change points can then be used to

---

4 See Dukes, Bowlin and MacDonald; 1987; Lunde and Timmermann, 2002; Woodward and Anderson, 2003; Pagan and Sossounov, 2003.
divide the stock index return series into n sub-periods. Categorization of these sub-periods into bear, bull and ‘other’ (non-bull and non-bear) states can be done ex post (after the regimes have been identified) by comparing the mean of the stock index returns in each period to critical threshold values.

To present the main idea of the penalized LSE technique, we begin with a simple model where only one change point exists in the mean of the return series. The general results with multiple breaks will be stated later. Equations A.1 to A3 are used to detect the start and end points of the daily return series we analyze.

Assume that the return series \( R_t \) has the following structure:

\[
\begin{align*}
R_t &= \mu_1 + \sigma_1 \varepsilon_t & \text{if} & & 1 \leq t \leq k_1 - 1 \\
R_t &= \mu_2 + \sigma_2 \varepsilon_t & \text{if} & & k_1 \leq t \leq T
\end{align*}
\]  

(A.1)

where \( T \) is the sample size, \( k_1 \) is the unknown break point, \( \mu_1 \) and \( \sigma_1 \) is the mean and volatility (respectively) of the first \((1 \text{ to } k_1-1)\) return observations, while \( \mu_2 \) and \( \sigma_2 \) is the mean and volatility (respectively) of the last \((k_1 \text{ to } T)\) return observations. \( \varepsilon_t \) is White noise. The sum of squared residuals can be calculated using the following formula:

\[
S_f (l = 1) = \sum_{i=1}^{k_1-1} (R_i - \hat{\mu}_1)^2 + \sum_{i=k_1}^{T} (R_i - \hat{\mu}_2)^2
\]  

(A.2)

where ‘\( l \)’ represents the number of change points.

The optimal break point \( \hat{k}_1 \) can be obtained by minimizing the sum of squared residuals among all possible sample splits.

\[
\hat{k}_1 = \arg \min_{1 \leq l \leq T-1} S_f (l = 1)
\]  

(A.3)

For the multiple breaks case, the model becomes:

\[
R_t = \mu_1 + \sigma_1 \varepsilon_t & \text{if} & & 1 \leq t \leq k_1 - 1
\]
\[ R_t = \mu_2 + \sigma_2 \varepsilon_t \quad \text{if} \quad k_1 \leq t \leq k_2 - 1 \]

\[ R_t = \mu_m + \sigma_m \varepsilon_t \quad \text{if} \quad k_m \leq t \leq T \]

(A.4)

where: \( k_2 \) is change point number 2

\( \mu_2 \) is the mean of the \((k_1 \text{ to } k_2 - 1)\) return observations

\( \sigma_2 \) is the volatility of the \((k_1 \text{ to } k_2 - 1)\) return observations

\( k_m \) is change point number \( m \)

\( \mu_m \) is the mean of the last \((k_m \text{ to } T)\) return observations

\( \sigma_m \) is the volatility of the last \((k_m \text{ to } T)\) return observations

The sum of squared residuals is:

\[ S_f (l = m) = \sum_{t=k_1}^{k_1 - 1} (R_t - \hat{\mu}_1)^2 + \sum_{t=k_2}^{k_2 - 1} (R_t - \hat{\mu}_2)^2 + \ldots + \sum_{t=k_m}^{T} (R_t - \hat{\mu}_m)^2 \]

(A.5)

The optimal break point estimators are defined as:

\[ (\hat{k}_m) = \arg \min_{1 \leq k_1 < \ldots < k_m \leq T} S_f (l = m) \]

(A.6)

The least-squared estimation procedure always favors the multiple structural change point models over the single or no change point model [ \( \min S_f (l = m) < \ldots < \min S_f (l = 1) < S_f (l = 0) \) ]. However, the more complex the models are, the less reliable the change point estimates become. The model order decision criterion used in this study is the Bayesian Information Criterion (BIC) of Schwarz (1978).\(^5\) The BIC selects among models by weighting the trade-off between increased information and decreased reliability. Using the BIC procedure, the optimal number of change points (\( l \)) can be estimated by maximizing the following function:

\(^5\) The model order decision criterion or selection criterion used in this study was originally based on the one employed by Kühn (1999). We have imposed a few simplifying assumptions so that the criterion of Kühn produces effectively similar results to that of Schwarz (1978).
\[ BIC(l) = -\frac{T}{2} \log \left( \frac{S_r(l)}{T} \right) - l \log(T) \]  

(A.7)

Here:

\[ \left[ -\frac{T}{2} \log \left( \frac{S_r(l)}{T} \right) \right] = \text{maximized log-likelihood} \]

\[ [\log(T)] = \text{penalty term} \]

We analyze three different return indices: 1) SPPR equal-weighted return index (EW), 2) SPPR value-weighted return index (VW), 3) All Ordinaries Accumulation return index (XOA-A). The first two series were obtained from the Share Price and Price Relative Data Base (SPPR). They are constructed using the stock returns of all Australian listed and previously listed companies on the Australian market from January 1974 to December 2001. The All Ordinaries Accumulation Index covers a period beginning January 1980 and ending December 2001, and is taken from the Core Research Data. Differences between the sample periods of the XOA-A and the EW and VW indices imply that the results of the bear market classification procedures using these three return indices will only be comparable from January 1980 to December 2001. Consequently, we restrict our sample period to this time frame.

The location of the optimal change points identified by the penalized LSE allows us to pinpoint the start of the stock market regimes in the three return series. Table A1 presents descriptive statistics for the market regimes. Given that we do not have priors, we will assume that a bear market exists either when mean returns are statistically significantly negative or when mean returns are significantly below the risk-free rate. We use two critical threshold values as it could be argued that investors are interested in realizing not just positive returns, but returns in excess of the risk-free rate. Irrespective of the method used to assess statistical significance, we are able to detect two bear periods in the equal-
weighted return series (and none in the VW and XOA-X series.) These are the November 1987 to February 1988 and April 2000 to May 2000 market regimes. According to the figures in Table A2, the November 1987 to February 1988 bear period lasted for 4 months, during which investors lost on average 6.88% of the value of their investment each month. The April 2000 to May 2000 bear period had a shorter duration (2 months) but with more serious consequences for investors. Over this time window, the average monthly return in the EW index was -13.45%.

References


Kühn, C. 1999, “An estimator of the number of change points based on a weak invariance principle”, Munich University of Technology, Centre for Mathematical Sciences.


Since t-statistics cannot be calculated for regimes with a duration of 1 month, we impose the restriction of a minimum 2-month duration for bear markets.


Table A1: Stock Market Regimes in the Return Series.

This table presents descriptive statistics for the stock market regimes identified in the equal-weighted (EW), value-weighted (VW) and the All Ordinaries Accumulation (XOA-X) monthly return series. The starting months of the regimes are the optimal change points identified by the penalised LSE approach described in Appendix A. The duration of the regimes is expressed in months, while the mean monthly returns and standard deviations are in percentages. The table also presents two t-statistics for each regime. The conventional t-statistics (t) is computed by dividing the mean (monthly) returns of the regimes (Column 4) by the corresponding standard deviations (Column 5). It tests whether the mean returns of the regimes are statistically significantly different from zero. $t^*$ is computed by subtracting the average 1-month risk free rate ($R_f$) over the regimes’ duration (Column 6) from the mean returns of the regimes (Column 4), before dividing it by the corresponding standard deviations (Column 5). It tests whether the mean returns of the regimes are statistically significantly different from the risk-free rate. The risk-free rate is proxied by the monthly rates on the 13 week Treasury notes and is obtained from the SPPR Database.

<table>
<thead>
<tr>
<th>Series</th>
<th>Market Regimes</th>
<th>Duration (months)</th>
<th>Mean (%)</th>
<th>Std. Dev. (%)</th>
<th>$R_f$</th>
<th>t</th>
<th>$t^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>EW</td>
<td>Jan. 80 – Jun. 82</td>
<td>30</td>
<td>1.76</td>
<td>6.47</td>
<td>1.01</td>
<td>0.27</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>Jul. 82 - Jul. 86</td>
<td>49</td>
<td>3.28</td>
<td>4.88</td>
<td>1.01</td>
<td>0.67</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>Aug. 86 - Sep. 87</td>
<td>14</td>
<td>6.46</td>
<td>4.6</td>
<td>1.14</td>
<td>1.4</td>
<td>1.17</td>
</tr>
<tr>
<td></td>
<td>Oct. 87</td>
<td>1</td>
<td>-29.33</td>
<td>-</td>
<td>0.83</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Nov. 87 - Feb. 88</td>
<td>4</td>
<td>-6.88</td>
<td>1.64</td>
<td>0.81</td>
<td>-4.2**</td>
<td>-4.69***</td>
</tr>
<tr>
<td></td>
<td>Mar. 88 - Jun. 89</td>
<td>16</td>
<td>0.14</td>
<td>3.84</td>
<td>1.09</td>
<td>0.04</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>Jul. 89 - Aug. 89</td>
<td>2</td>
<td>11.89</td>
<td>2.77</td>
<td>1.31</td>
<td>4.29***</td>
<td>3.82***</td>
</tr>
<tr>
<td></td>
<td>Sep. 89 – Jan. 91</td>
<td>17</td>
<td>-1.84</td>
<td>2.79</td>
<td>1.16</td>
<td>-0.66</td>
<td>-1.08</td>
</tr>
<tr>
<td></td>
<td>Feb. 91 - Feb. 92</td>
<td>13</td>
<td>6.7</td>
<td>4.59</td>
<td>0.75</td>
<td>1.46</td>
<td>1.3</td>
</tr>
<tr>
<td></td>
<td>Mar. 92 – Nov. 92</td>
<td>9</td>
<td>0.25</td>
<td>2.52</td>
<td>0.49</td>
<td>0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>Dec. 92 – Feb. 94</td>
<td>15</td>
<td>8.04</td>
<td>5.42</td>
<td>0.41</td>
<td>1.48</td>
<td>1.41</td>
</tr>
<tr>
<td></td>
<td>Mar. 94 – Jun. 95</td>
<td>16</td>
<td>-1.72</td>
<td>3.36</td>
<td>0.54</td>
<td>-0.51</td>
<td>-0.67</td>
</tr>
<tr>
<td></td>
<td>Jul. 95 – Oct. 98</td>
<td>40</td>
<td>0.98</td>
<td>5.12</td>
<td>0.49</td>
<td>0.19</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Nov. 98 – Mar. 00</td>
<td>16</td>
<td>5.59</td>
<td>4.7</td>
<td>0.4</td>
<td>1.19</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>Apr. 00 – May 00</td>
<td>2</td>
<td>-13.45</td>
<td>3.8</td>
<td>0.48</td>
<td>-3.54***</td>
<td>-3.67***</td>
</tr>
<tr>
<td></td>
<td>Jun. 00 – Sep. 01</td>
<td>16</td>
<td>-1.21</td>
<td>5.66</td>
<td>0.45</td>
<td>-0.21</td>
<td>-0.29</td>
</tr>
<tr>
<td>VW</td>
<td>Jan. 80 – May 87</td>
<td>89</td>
<td>1.99</td>
<td>5.7</td>
<td>1.03</td>
<td>0.35</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Jun. 87 – Sep. 87</td>
<td>4</td>
<td>5.73</td>
<td>6.25</td>
<td>0.94</td>
<td>0.92</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>Oct-87</td>
<td>1</td>
<td>-39.54</td>
<td>-</td>
<td>0.83</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Nov. 87 – Dec. 01</td>
<td>170</td>
<td>0.9</td>
<td>3.59</td>
<td>0.63</td>
<td>0.25</td>
<td>0.08</td>
</tr>
<tr>
<td>XOA-X</td>
<td>Jan. 80 – Sep. 87</td>
<td>93</td>
<td>2.11</td>
<td>5.18</td>
<td>1.03</td>
<td>0.41</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>Oct-87</td>
<td>1</td>
<td>-15.55</td>
<td>-</td>
<td>0.83</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Nov-87</td>
<td>1</td>
<td>-31.98</td>
<td>-</td>
<td>0.82</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Dec 87 – Jan. 94</td>
<td>74</td>
<td>1.21</td>
<td>3.86</td>
<td>0.84</td>
<td>0.31</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>Feb 94 – Jan. 95</td>
<td>12</td>
<td>-1.11</td>
<td>1.92</td>
<td>0.49</td>
<td>0.58</td>
<td>-0.83</td>
</tr>
<tr>
<td></td>
<td>Feb. 95 – Dec. 01</td>
<td>83</td>
<td>1.04</td>
<td>2.83</td>
<td>0.47</td>
<td>0.37</td>
<td>0.2</td>
</tr>
</tbody>
</table>

*,**,*** denote significance at the 10%, 5% and 1% significance level, respectively.